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SPONTANEOUS BREAKING OF SCALE INVARIANCE IN A

SUPERSYMMETRIC MODEL

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ABSTRACT

The phase structure of a large N, O(N) supersymmetric model in three dimensions is studied. Of special interest is the spontaneous breaking of scale invariance which occurs at a fixed value of the coupling constant $\lambda_o = \lambda_c = 4\pi$. In this phase the bosons and fermions acquire a mass while a goldstone boson (dilaton) and goldstone fermion ("dilatino") are dynamically generated as massless bound states. The absence of renormalization of the dimensionless coupling constant λ_o leaves these goldstone particles massless.

INTRODUCTION

Dynamical generation of mass scales is a long standing problem in quantum field theory. The interest in this issue has been enhanced by the attempt to understand the very different orders of magnitude of mass scales that appear in particle physics. Hope to unveil this mystery came from super-symmetry; consequently, one of the most interesting issues in super-symmetric theories is their ultraviolet behavior and phase structure. Though in general we lack exact solutions, much of the theory's secrets are often revealed in simple models and useful approximations. Such a nonperturbative and systematic approach, is provided by the large N expansion in quantum field theory where, in many cases, the leading order exhibits the true dynamical effects of the theory. Indeed, in a realistic theory the leading order in $1/N$ exhibits some of the known dominant phenomenological features; (i.e. Zweig's rule and Regge phenomenology represented in the planar diagrams in QCD). The large N expansion is employed in the present paper in order to study an $O(N)$ symmetric, supersymmetric theory, where the leading $1/N$ structure is exactly calculable.

A new phase structure for the large N $\gamma(\vec{\phi}^2)^3$ theory in three dimensions has been recently found.[1] At the tricritical point, where all renormalized dimensional parameters vanish ($\rho\vec{\phi}^2$ and $\lambda(\vec{\phi}^4)^2$), the theory is determined by a renormalizable scale invariant lagrangian. At large N the theory has a nontrivial ultraviolet fixed point associated with a dynamical mass, generated for the $\vec{\phi}$ particles through dimensional transmutation. Scale invariance is thus, spontaneously broken, and a

goldstone boson of broken scale invariance appears as a $\vec{\phi}\text{-}\vec{\phi}$ bound state. This goldstone boson, the dilaton , is massless in leading $1/N$ since no perturbative renormalization of the coupling η exists at this order, and, therefore, no explicit breaking of scale invariance exists. The large N leading perturbative β function of the theory is identically zero (next order in $(1/N)$ has been calculated[2]) whereas a nonperturbative leading $O(1)$ contribution to the β function possesses the ultraviolet fixed point mentioned above.[3]

In the present paper we establish an interesting new phase structure for a supersymmetric theory in three dimensions, which is reminiscent of the one found for the $\eta(\vec{\phi}^2)^3$ theory. At large N we find spontaneous breaking of scale invariance and a dynamical mass generated for the scalars and fermions in the supersymmetric ground state and thus the formation of the goldstone mode. The dilaton and the "dilatino" appear as massless bound states in the scalar-scalar and scalar-fermion scattering amplitudes, respectively.

In Sec. 2 we present our variational calculation following the method used in Ref. 4 (where it was employed to show that large N $\lambda(\vec{\phi}^2)^2$ in four dimensions is unstable). The new ingredient here is the fermion contribution to the Hartree-Fock ground state energy. The variational wave functional is the direct product of scalar and fermion components. This calculation gives the exact answer at $N\to\infty$. In Sec. 3 we find the gap equations that determine the scalar and fermion masses. The phase structure is then revealed and summarized in Figs 2 and 3. Scale invariance is spontaneously broken at $\mu^2 = 0$, if the coupling constant λ_0 takes exactly the value $\lambda_0 = 4\pi$. In Sec. 4 we calculate in leading $1/N$ the scattering amplitudes and find the expected dilaton and "dilatino" poles in the $\mu^2 = 0$, $\lambda_0 = 4\pi$ goldstone phase. Sec. 5 summarizes our conclusions.

II. <u>VARIATIONAL METHODS</u>

We will discuss a model in three space time dimensions given by the action

$$S = \int d^3x \; d^2\theta \; \mathcal{L}(x, \theta) \tag{2.1}$$

where

$$\mathcal{L}(x, \theta) = \tfrac{1}{4} \; \overline{D \vec{\Phi}} \cdot D\vec{\Phi} \; + \; \tfrac{\lambda_0}{4N} \left(\vec{\Phi}^2 + N \tfrac{\mu_0}{\lambda_0} \right)^2 \tag{2.2}$$

$\vec{\Phi}$ is an N component superfield

$$\vec{\Phi}(x, \theta) = \vec{A}(x) + \bar{\theta} \, \vec{\Psi}(x) + \tfrac{1}{2} \, \bar{\theta}\theta \, \vec{F}(x) \tag{2.3}$$

$\vec{A}(x), \vec{\Psi}(x)$, and $\vec{F}(x)$ are the real scalar, two component majorana and auxiliary fields, respectively. In Eq. 2.2, D is the super covariant derivative $D = \tfrac{\partial}{\partial \bar{\theta}} - i \not{\partial} \theta$ with θ a two component majorana anticommuting variable. Written in terms of component fields, the action is

$$S = \int d^3x \left\{ \tfrac{1}{2} \left(\partial_\mu \vec{A} \right)^2 + \tfrac{1}{2} \vec{\Psi} \, i \not{\partial} \vec{\Psi} + \tfrac{1}{2} \vec{F}^2 \right.$$
$$+ \tfrac{\mu_0}{2} \left(2 \vec{A} \cdot \vec{F} - \vec{\Psi} \cdot \vec{\Psi} \right) \tag{2.4}$$
$$\left. + \tfrac{\lambda_0}{4N} \left[2 \vec{A}^2 (2 \vec{A} \cdot \vec{F} - \vec{\Psi} \cdot \vec{\Psi}) - 4(\vec{A} \cdot \vec{\Psi})(\vec{A} \cdot \vec{\Psi}) \right] \right\}$$

Integrating out the $\vec{F}(x)$ component auxiliary field

$$\vec{F} + \mu_0 \vec{A} + \tfrac{\lambda_0}{N} (\vec{A}^2) \vec{A} = 0 \tag{2.5}$$

gives finally the lagrangian

$$\mathcal{L}_{(x)} = \frac{1}{2}[(\partial_\mu \vec{A})^2 - \mu_o^2 \vec{A}^2] + \frac{1}{2}\vec{\Psi}(i\not\partial - \mu_o)\vec{\Psi}$$

$$- \frac{\lambda_o}{N}\mu_o(\vec{A}^2)^2 - \frac{\lambda_o^2}{2N^2}(\vec{A}^2)^3 \tag{2.6}$$

$$- \frac{\lambda_o}{2N}\left[\vec{A}^2(\vec{\Psi}\cdot\vec{\Psi}) + 2(\vec{A}\cdot\vec{\Psi})(\vec{A}\cdot\vec{\Psi})\right]$$

In the large N limit we hold λ_o, μ_o fixed as $N\to\infty$. Standard technique of summing the infinite number of cactus diagrams use Euclidean functional integrals[5] or Hamiltonian methods[4,6]. Below, we use the variational method of Refs. 4 and find the best plane wave ground state which is represented by the wave functional

$$\tilde{\Psi}\{\vec{A}, \vec{\Psi}\} = \Psi\{\vec{A}\}\,\Psi\{\vec{\Psi}\} \tag{2.7}$$

where

$$\Psi\{\vec{A}\} = \exp\left\{-\int d^3x\, d^3y\, (\vec{A}_{(x)} - \vec{A}_c)G_{m_A}(x-y)(\vec{A}_{(y)} - \vec{A}_c)\right\} \tag{2.8a}$$

$$\Psi\{\Psi\} = \exp\left\{-\int d^3x\, d^3y\, \vec{\Psi}_{(x)}\Delta_{m_\Psi}(x-y)\vec{\Psi}_{(y)}\right\}|0\rangle_o \tag{2.8b}$$

G_{m_A} and Δ_{m_Ψ} are the boson and fermion correlation functions. The background field $\vec{A}_c$ as well as the boson and fermion masses in the correlation functions will be determined by minimizing the Hartree-Fock ground state energy. It has been shown in Refs. 4 that this approach is equivalent to an effective potential calculation as long as the gap equation is satisfied. End point solutions for global minimum of the ground state energy are obvious in this variational calculation, but missed altogether in the effective potential approach.[7] In the present analysis , gap equation solutions will play the main role and thus our results can be rederived also in a standard large N effective potential calculation.

The Hartree-Fock ground state energy, to leading order in N, is given by the sum of kinetic and potential energies and

$$W(m_A, m_\psi, \vec{A}_c) = K_A(m_A, \vec{A}_c) + K_\psi(m_\psi) + \langle V(\vec{A}, \vec{\psi}) \rangle_0 \qquad (2.9)$$

can be calculated using a trial plane wave expansion[4] of the quantum operators $\vec{A}(x)$ and $\vec{\psi}(x)$ or equivalently through a Euclidean path integral. One finds for the boson kinetic energy,

$$\frac{\partial}{\partial m_A^2} K_A = -\frac{m_A^2}{2} \frac{\partial}{\partial m_A^2} \langle \vec{A}^2 \rangle \qquad (2.10)$$

and for the fermion kinetic term, we use the relation

$$\int D\psi \, e^{-\int d^3x \, \mathcal{L}_E(x)} = e^{-[K_\psi(m_\psi) + \frac{m_\psi}{2} \langle \vec{\psi} \cdot \vec{\psi} \rangle] VT} \qquad (2.11)$$

where $\mathcal{L}_E = \frac{1}{2} \vec{\psi}(\partial\!\!\!/ + m_\psi)\vec{\psi}$ and find

$$\frac{\partial}{\partial m_\psi} K_\psi = -\frac{m_\psi}{2} \frac{\partial}{\partial m_\psi} \langle \vec{\psi} \cdot \vec{\psi} \rangle \qquad (2.12)$$

We distinguish between the normal ordered masses m_A and m_ψ of the boson and fermion fields and will let the minimalization procedure to choose a supersymmetric or non-supersymmetric ground state.

The vacuum expectation value of an $O(N)$ singlet quantum operator at large N satisfies[8] $\langle Q_N^k(x) \rangle = \langle Q_N(x) \rangle^k + O(\frac{1}{N})$ and thus the potential energy from Eqs. 2.6 and 2.9 is given to leading order in N by

$$\langle V(\vec{A}, \vec{\psi}) \rangle_0 = \frac{\mu_0^2}{2} \langle \vec{A}^2 \rangle + \frac{\mu_0}{2} \langle \vec{\psi} \cdot \vec{\psi} \rangle + \frac{\lambda_0 \mu_0}{N} \langle \vec{A}^2 \rangle^2$$

$$+ \frac{\lambda_0^2}{2N^2} \langle \vec{A}^2 \rangle^3 + \frac{\lambda_0}{2N} \langle \vec{A}^2 \rangle \langle \vec{\psi} \cdot \vec{\psi} \rangle \qquad (2.13)$$

The ground state energy in Eq. 2.9 is determined in leading order in N by the quantum fluctuation of $\vec{A}^2$ and $\vec{\Psi} \cdot \vec{\Psi}$, namely

$$\langle \vec{A}^2 \rangle = N \vec{A}_c^2 + N \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{k^2 + m_A^2} \right)$$

(2.14a)

$$= N \vec{A}_c^2 + \frac{N}{2\pi^2} \left[\Lambda_A - \frac{\pi}{2} |m_A| \right]$$

and

$$\langle \vec{\Psi} \cdot \vec{\Psi} \rangle = N \int \frac{d^3k}{(2\pi)^3} \left(\frac{-2 m_\Psi}{k^2 + m_\Psi^2} \right)$$

(2.14b)

$$= -N \frac{m_\Psi}{\pi^2} \left(\Lambda_\Psi - \frac{\pi}{2} |m_\Psi| \right)$$

Λ_A and Λ_Ψ are ultraviolet cutoffs which we will take now to be equal in order not to introduce an explicit breaking of supersymmetry.

From Eqs. (2.10), (2.12), and (2.14a,b) one finds (we take $m_A > 0$)

$$k_A(m_A) = \frac{N}{24\pi} m_A^3$$

(2.15a)

and

$$k_\Psi(m_\Psi) = \frac{N}{4\pi^2} \Lambda m_\Psi^2 - \frac{N}{6\pi} |m_\Psi|^3$$

(2.15b)

The Hartree-Fock ground state energy in Eq. 2.9 is now given by

$$\frac{1}{N} W(m_A, m_\psi, \vec{A}_c, \Lambda) = \frac{\Lambda m_\psi^2}{4\pi^2} - \frac{|m_\psi|^3}{6\pi} + \frac{m_A^3}{24\pi}$$

$$+ \frac{\mu_o^2}{2}\left(\vec{A}_c^2 + \frac{\Lambda}{2\pi^2} - \frac{m_A}{4\pi}\right) + \frac{\mu_o}{2}\left(-\frac{\Lambda}{\pi^2} m_\psi + \frac{m_\psi |m_\psi|}{2\pi}\right) \qquad (2.16)$$

$$+ \lambda_o \mu_o\left(\vec{A}_c^2 + \frac{\Lambda}{2\pi^2} - \frac{m_A}{4\pi}\right)^2 + \frac{\lambda_o^2}{2}\left(\vec{A}_c^2 + \frac{\Lambda}{2\pi^2} - \frac{m_A}{4\pi}\right)^3$$

$$+ \frac{\lambda_o}{2}\left(\vec{A}_c^2 + \frac{\Lambda}{2\pi^2} - \frac{m_A}{4\pi}\right)\left(-\frac{\Lambda m_\psi}{\pi^2} + \frac{m_\psi |m_\psi|}{2\pi}\right)$$

The only renormalization required in the leading order in N is

$$\mu = \mu_o + \lambda_c\left(\frac{\Lambda}{2\pi^2}\right), \qquad \lambda = \lambda_o \qquad (2.17)$$

After some rearrangement of Eq. 2.16, one finds

$$\frac{8\pi}{N} W \equiv \omega(m_A, m_\psi, \vec{A}_c^2, \Lambda)$$

$$= \frac{1}{3}\left(m_A^3 - |m_\psi|^3\right) \qquad (2.18)$$

$$+ \left(2\frac{\Lambda}{\pi} - |m_\psi|\right)\left[m_\psi - \mu - \lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right)\right]^2$$

$$+ \left(|m_\psi| - m_A + 4\pi \vec{A}_c^2\right)\left[\mu + \lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right)\right]^2$$

We notice that the renormalized ω is not cutoff independent for arbitrary values of the variational parameters, $m_\psi, m_A, \vec{A}_c$. However, we have not yet applied the minimalization procedure to choose the best plane wave gound state.[4] This will be carried out in the next section.

III. GAP EQUATION AND PHASE STRUCTURE

At its minimum, the Hartree-Fock ground state energy satisfies the extremum conditions $\partial W/\partial m_\psi = 0$ and $\partial W/\partial m_A = 0$. The first relation implies

$$\frac{\partial W}{\partial m_\psi} = 0 \longrightarrow \left(\frac{\Lambda}{\pi} - |m_\psi|\right)\left(m_\psi - \mu - \lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right)\right) = 0 \qquad (3.1)$$

which is the gap equation for the fermion mass (Fig. 1a). Since the cutoff dependent term in Eq. 2.18 is proportional to the fermion gap equation [we neglect the uninteresting $m_\psi \sim 0(\Lambda)$ solution in Eq. 3.1], we see that if $m_\psi = \mu + \lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right)$ then $W(m_A, m_\psi, \vec{A}_c^2, \Lambda)$ is now cutoff independent. Namely, if one inserts now the fermion gap equation solution into Eq. 2.18, one finds the vacuum energy,

$$W(m_A, m_\psi, \vec{A}_c^2) = \frac{1}{3}\left(m_A^3 - |m_\psi|^3\right) \qquad (3.2)$$
$$+ m_\psi^2\left(|m_\psi| - m_A + 4\pi\vec{A}_c^2\right)$$

The boson gap equation (Fig. 1b)

$$\frac{\partial W}{\partial m_A} = 0 \longrightarrow m_A^2 + \lambda\left(\frac{\Lambda}{\pi^2}m_\psi - \frac{m_\psi|m_\psi|}{2\pi}\right)$$
$$-\left[\mu + \lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right)\right]\left[\mu + 3\lambda\left(\vec{A}_c^2 - \frac{m_A}{4\pi}\right) + \lambda\frac{\Lambda}{\pi^2}\right] = 0 \qquad (3.3)$$

can be written now as

$$m_A^2 = m_\psi^2 + 2\lambda\, m_\psi\left(\vec{A}_c^2 - \frac{m_A}{4\pi} + \frac{|m_\psi|}{4\pi}\right) \qquad (3.4)$$

One notices that W in Eq. 3.2 can be written as

$$W = \frac{m_A^3}{3}\left(1 + 2R^3 - 3R^2\right) + 4\pi m_\psi^2\vec{A}_c^2 \qquad (3.5)$$

where $R = |m_\psi|/m_A$. Since both terms in Eq. 3.5 are never negative, the absolute minimum of the ground state energy can be found by separately minimizing each term. The first term is zero at its minimum value where $R = 1$. The vacuum energy becomes $W = 4\pi m_\psi^2 \vec{A}_c^2$ which has two possible minima. For $|m_\psi| = m_A \neq 0$ and $\vec{A}_c^2 = 0$, we have an $O(N)$ symmetric, supersymmetric ground state. Alternately, if $|m_\psi| = m_A = 0$ with $\vec{A}_c^2 \neq 0$, the $O(N)$ symmetry is broken in the ground state. Both solutions imply $W_{min} = 0$ because the supersymmetry always remains unbroken at the minimum.

To study the vacuum structure we examine the gap equations. Since $|m_\psi| = m_A$ it is sufficient to study the fermion gap equation obtained from Eq. 3.1,

$$m_\psi = \mu + \lambda \left(\vec{A}_c^2 - |m_\psi|/4\pi \right) \qquad (3.6)$$

This simple relation, when combined with Eq. 3.5, reveals a very interesting phase structure. We normally would expect to find two possible phases, an $O(N)$ symmetric phase for $\mu > 0$ and an $O(N)$ broken symmetry phase for $\mu < 0$. Indeed we find these two conventional solutions to the gap equation.

$$a) \; \mu > 0 : \; \vec{A}_c = 0 \; , \quad m_\psi = \mu/(1 + \lambda/4\pi) > 0$$

$$(3.7)$$

$$b) \; \mu < 0 : \; m_\psi = 0, \quad \vec{A}_c^2 = -\mu/\lambda > 0$$

However, there exists a totally new class of solutions to the gap equation which produces an $O(N)$ symmetric phase different from case (a) for certain ranges of the renormalized parameters.

$$c1) \; \mu > 0 \; , \; \lambda > 4\pi : \; \vec{A}_c = 0, \quad m_\psi = -\mu/(\lambda/4\pi - 1) < 0$$

$$c2) \; \mu = 0, \; \lambda = 4\pi : \; \vec{A}_c = 0, \; m_\psi \; \text{arbitrary} < 0$$

(3.8)

$$c3) \; \mu < 0, \; \lambda < 4\pi : \; \vec{A}_c = 0, \; m_\psi = \mu/(1 - \lambda/4\pi) < 0$$

Note we have chosen $\lambda > 0$ by convention.

The different phases of the model are summarized in Fig. 2. In case (a), the O(N) symmetry is preserved and the fermion mass is positive (note that there is no trivial γ_5 symmetry to change the sign of the fermion mass term for two component fermions in three dimensions). This is the expected symmetric phase for positive mass, μ. In case (b), the O(N) symmetry is spontaneously broken and the bosons are massless goldstone particles. Supersymmetry implies that the fermionic partners to these bosons are also massless. This is the expected broken symmetry phase for negative mass, μ. Case (c) is the new O(N) symmetric phase which exists for both signs of the mass parameter, μ; but for a restricted range of couplings, λ. The fermion mass is negative for this phase.

An alternate description of the phase structure of the model is given in Figure 3. We show lines of "constant physics" as function of the renormalized parameters, μ and λ. For the symmetric phases, (a) and (c), the lines correspond to constant physical mass for the fermions and bosons. Note that the lines for phase (c) are all continuous through the point, ($\mu = 0$, $\lambda = 4\pi$). For the broken symmetry phase, the line corresponds constant vacuum expectation value of the scalar field, $\vec{A}_c$. In regions II and IV, the normal phases provide the unique ground state. However in regions I and III, the normal phases are degenerate with the new phase. At each point of region I, there are two possible ground states, both O(N) symmetric and

supersymmetric, but having different values for the particle masses. At each point of region III, an $O(N)$ symmetric ground state is degenerate with states where the $O(N)$ symmetry is spontaneously broken. The degeneracy is exact as the unbroken supersymmetry guarantees that the vacuum energy vanishes for all ground states.

The point $\mu = 0$, is of particular interest. Since μ is the only dimensional parameter in the theory, the lagrangian is scale invariant if $\mu = 0$. (We consider, as above, the theory at $\partial W/\partial m_\psi = 0$, namely the fermion gap equation is satisfied, and thus the Hartree-Fock ground state energy W has no cutoff dependence). At leading N, the coupling constant λ is not renormalized. The gap equation (Eq. 3.6) has now the trivial solution $m_\psi = 0$, $\tilde{A}_c = 0$ that give $W = 0$. This is the case where we are exactly at the critical point of the theory and is analogous to the $m = 0$ solution at the tricritical point in Refs. 1,2.

There is, however, another possibility if the coupling constant λ takes exactly the value $\lambda = 4\pi$. In this case, $m_\psi < 0$ can take any value (while $\tilde{A}_c^2 = 0$). This gives a supersymetric ($|m_\psi| = m_A$), $O(N)$ symmetric phase in which by dimensional transmutation, the coupling λ is traded for a dynamically generated mass. Scale invariance of the original lagrangian is spontaneously broken due to the appearance of $|m_\psi| = m_A \neq 0$. Note that all lines of phase (c) pass through the point, ($\mu = 0$, $\lambda = 4\pi$). In the next section, we discuss the appearance of a goldstone boson (dilaton) and a goldstone fermion ("dilatino") associated with this phase.

IV. THE DILATON AND DILATINO POLES

As found at the end of Section III, the theory with $\mu = 0$ has, in addition to the $m = \vec{A}_c^2 = 0$ critical point minimum, also a massive phase in which scale invariance is spontaneously broken. Since this ground state is supersymmetric, the goldstone boson associated with the spontaneous breaking, the dilaton, is accompanied by its supersymmetric partner, a goldstone fermion, we call a "dilatino".

The dilatino $p^2 = 0$ pole should be found in the fermion-boson scattering amplitude. Indeed, the leading order in N (Fig. 4) gives as seen from Eq. 2.6

$$T_{\psi_A, \psi_A}(p^2) = 2\frac{\lambda}{N}\left\{1 - 2\lambda \int \frac{d^3k}{(2\pi)^3}\left(\frac{\not{p} + \not{k} - m_\psi}{(p+k)^2 + m_\psi^2}\right)\left(\frac{1}{k^2 + m_A^2}\right)\right\}^{-1}$$

$$= \frac{2\lambda}{N}\left\{1 - \frac{\lambda}{4\pi}\int_0^1 d\alpha \frac{(1-\alpha)\not{p} - m_\psi}{[\alpha(1-\alpha)p^2 + \alpha m_\psi^2 + (1-\alpha)m_A^2]^{1/2}}\right\}^{-1}$$

(4.1)

In the supersymmetric ground state $|m_\psi| = m_A$ we have as $p^2 \to 0$

$$T_{\psi_A, \psi_A}(p^2) = 2\frac{\lambda}{N}\left\{1 + \frac{\lambda}{4\pi}\frac{|m_\psi|}{m_\psi} - \frac{\lambda}{2\pi}\frac{\not{p}}{|m_\psi|} + \dots\right\}^{-1}$$

(4.2)

and thus using Eq. 3.7, we find in the $m_\psi \neq 0$ solution of the $\mu = 0$ case, the dilatino pole (note that we use Euclidean metric)

$$T_{\psi_A, \psi_A}(p^2) = -\frac{16\pi}{N}\frac{|m_\psi|}{\not{p}}$$

(4.3)

The dilaton pole can be found, as in Ref.1, in the AA channel and here also in the $\bar{\psi}\psi$ channel. Fig. 5 gives the following relations for the scattering amplitudes

$$T_{11} = 4X + X B T_{11} + y F T_{21} \tag{4.4a}$$

$$T_{21} = 4y + y B T_{11} \tag{4.4b}$$

$$T_{12} = 4y + X B T_{12} + y F T_{22} \tag{4.4c}$$

$$T_{22} = y B T_{12} \tag{4.4d}$$

where we denote by an index 1 and 2 an AA state and a $\bar{\psi}\psi$ state, respectively. x and y are the couplings

$$X = 2 \frac{\lambda}{N} \mu_o + 3 \frac{\lambda^2}{N^2} \langle \vec{A}^2 \rangle \tag{4.5a}$$

$$y = - \frac{\lambda}{2N} \tag{4.5b}$$

B and F are the boson and fermion bubble diagrams given by

$$B = -i \int d^3z \, e^{i p^2} \langle \vec{A}^2_{(o)} \, \vec{A}^2_{(z)} \rangle$$

$$= -2N \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m_A^2} \frac{1}{(p+k)^2 + m_A^2} = - \frac{N}{4\pi} \int_0^1 \frac{d\alpha}{[\alpha(1-\alpha)p^2 + m_A^2]^{\frac{1}{2}}} \tag{4.6}$$

and

$$F = -i \int d^3z \, e^{ipz} \langle \vec{\Psi} \cdot \vec{\Psi}_{(0)}, \; \vec{\Psi} \cdot \vec{\Psi}_{(z)} \rangle = -2N i \int \frac{d^3k}{(2\pi)^3} Tr\left[\frac{1}{\not{k} - m_\Psi} \cdot \frac{1}{\not{p} + \not{k} - m_\Psi} \right]$$

$$= -2N \int \frac{d^3k}{(2\pi)^3} \left[\frac{2k(k+p) - 2m_\Psi^2}{(k^2 + m_\Psi^2)(\, (p+k)^2 + m_\Psi^2)} \right] \tag{4.7}$$

$$= -2N \left\{ \frac{\Lambda}{\pi^2} - \frac{|m_\Psi|}{2\pi} - \frac{1}{8\pi} (4 m_\Psi^2 + p^2) \int_0^1 d\alpha \, \frac{1}{[\alpha(1-\alpha)p^2 + m_\Psi^2]^{\frac{1}{2}}} \right\}$$

From Eqs. 4.4 a-d, one finds

$$T_{A^2, A^2} \equiv T_{11} = \frac{4x + 4y^2 F}{1 - xB - y^2 BF} \tag{4.8a}$$

$$T_{A^2, \bar{\Psi}\Psi} \equiv T_{12} = \frac{4y}{1 - xB - y^2 BF} = T_{21} \equiv T_{\bar{\Psi}\Psi, A^2} \tag{4.8b}$$

$$T_{\bar{\Psi}\Psi, \bar{\Psi}\Psi} \equiv T_{22} = \frac{4y^2 B}{1 - xB - y^2 BF} \tag{4.8c}$$

All these scattering amplitudes have the common denominator

$$1 - xB - y^2 BF = 1 - \left(2 \frac{\lambda}{N} \mu_0 + 3 \frac{\lambda^2}{N^2} \langle \vec{A}^2 \rangle \right) B - \left(\frac{\lambda_0}{2N} \right)^2 BF \tag{4.9}$$

which vanish at $p^2 = 0$ in the spontaneously broken scale invariant phase ($\mu = 0$). This can be seen from Eqs. 4.6 and 4.7 as $p^2 \to 0$

$$B(p^2) \to -\frac{N}{4\pi} \frac{1}{m_A} \left(1 - \frac{1}{12} \frac{p^2}{m_A^2} \right) \tag{4.10a}$$

$$F(p^2) \longrightarrow -\frac{2N}{\pi} \left(\frac{\Lambda}{\pi} - |m_\psi| - \frac{1}{12} \frac{p^2}{|m_\psi|} \right) \qquad (4.10b)$$

and using Eq. 2.14a we find

$$1 - XB - y^2 BF \xrightarrow[p^2 \to 0]{} 1 - \left(\frac{\lambda}{4\pi}\right)^2 + \left(\frac{\lambda}{8\pi}\right)^2 \frac{p^2}{m^2} \qquad (4.11)$$

which as expected exhibits the dilaton pole at $p^2 = 0$ if $\lambda = 4\pi$.

V. SUMMARY

Using a Hartree-Fock variational calculation, we studied the phase structure of an O(N) supersymmetric model in three dimensions. This variational calculation gives the exact answer in the limit $N \rightarrow \infty$. Figures 2 and 3 summarize our main findings. The ground state of this model at large N was found to be always supersymmetric, whereas the O(N) internal symmetry can be broken (phase b) or unbroken (phases a and c), depending on the values of μ^2 and λ, the only parameters of the renormalized theory.

Of special interest is the phase denoted by c (in Figs. 2 and 3). In this phase when the renormalized mass parameter μ is zero the lagrangian represents a scale invariant theory (λ is dimensionless in d = 3). Dimensional transmutation takes place if $\lambda = 4\pi$ and a mass $|m_\gamma| = m_A$ is generated for the O(N) bosons ($\vec{\phi}$) and fermions ($\vec{\psi}$). Scale invariance is spontaneously broken at this value of λ and a massless goldstone boson (dilaton) and goldstone fermion ("dilatino") appear in the scattering amplitudes of Figures 4 and 5 as $\vec{\phi} \cdot \vec{\phi}$, $\vec{\psi} \cdot \vec{\psi}$ and $\vec{\phi} \cdot \vec{\psi}$ bound states. Normally these goldstone particles acquire mass due to explicit breaking of scale invariance brought in by renormalization. Here, however, since λ is not renormalized the dilaton and "dilatino" are exactly massless. This result is reminiscent of what was found[1] in $\eta (\vec{\phi}^2)^3$ in three dimensions where at the tricritical point, an ultra-violet fixed point in the β function at $\eta = \eta_c = (4\pi)^2$ stabilized

the ground state while scale invariance was spontaneously broken. We are unaware of other models where the goldstone particle of broken scale invariance stays massless. The implications of this on realistic models need futher study,

ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Fig. 1: (a) Fermion mass gap equation (b) Boson mass gap equation. (Dashed line denotes the fermion propagator; solid line denotes the boson propagator).

Fig. 2: Phase structure of the theory. We plot $|m_\psi| = |m_A|$ as a function of μ for fixed coupling $\lambda > 0$. Cases (a) and (c) are $O(N)$ symmetric while case (b) has the $O(N)$ symmetry spontaneously broken. We also plot the vacuum value of the scalar field for the three phases.

Fig. 3: The different phases of Figure 2. We plot lines of "constant physics." For the conventional $O(N)$ symmetric phase (a), the line is for constant particle mass $m_\psi = |m_A|$. For the conventional $O(N)$ broken symmetry phase, the line is for constant vacuum value of the scalar field, $\vec{A}_c$. We also show the lines corresponding to the new $O(N)$ symmetric phase (c) with $m_\psi = -|m_A|$. At $\mu = 0$, all the lines of phase (c) pass through the point $\lambda = 4\pi$ and scale invariance is dynamically broken.

Fig. 4 Fermion-boson scattering amplitude in leading order in $1/N$. (Solid line - boson, dashed line - fermion). A massless "dilatino" is found in this channel if $\mu^2 = 0$ and $\lambda = 4\pi$.

Fig. 5: Boson-boson and fermion-fermion scattering amplitude in which a massless dilation appears (if $\mu^2 = 0$ and $\lambda = 4\pi$).
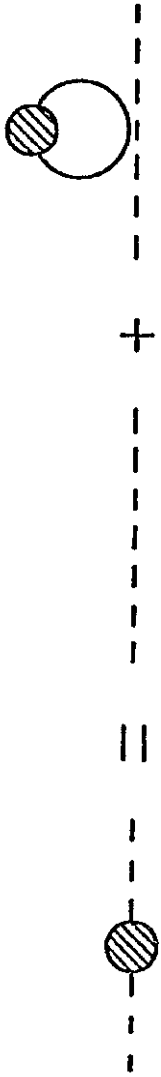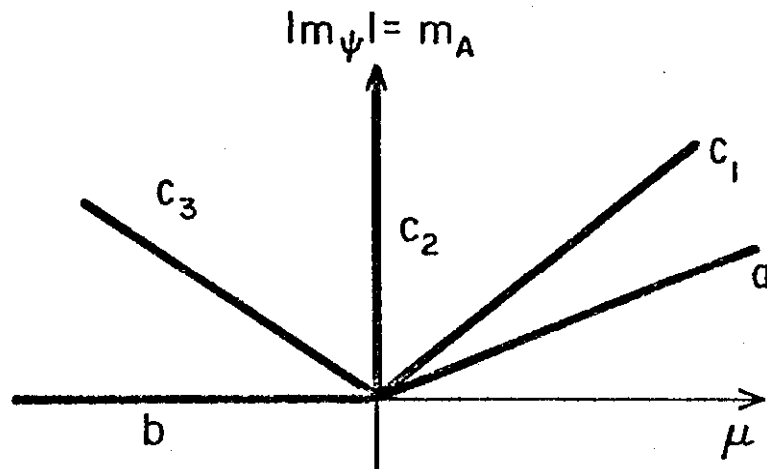
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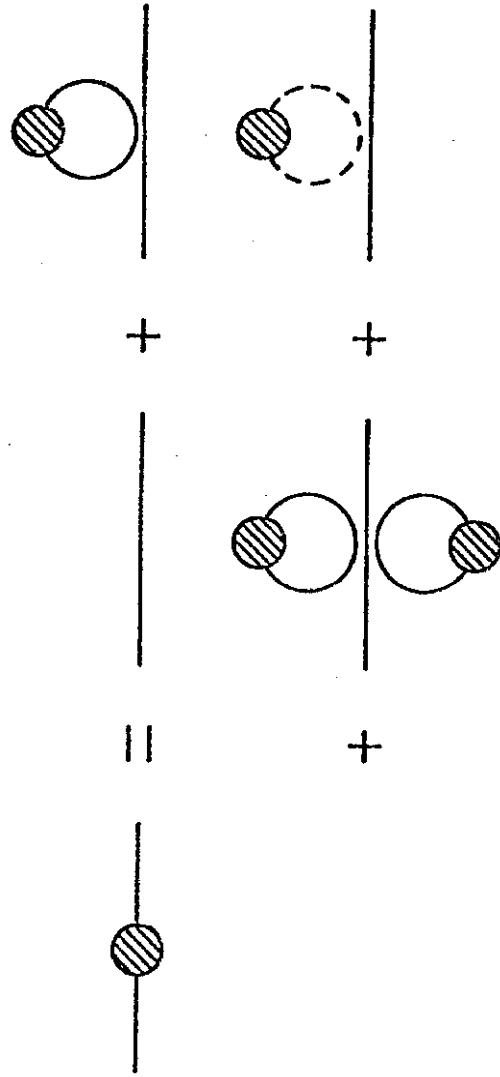
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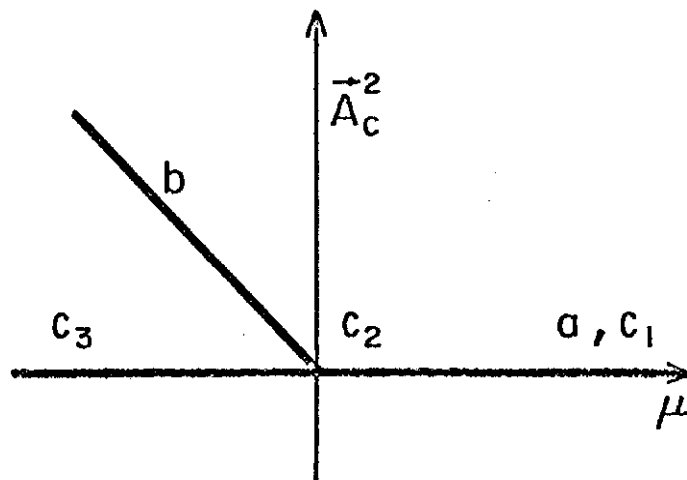
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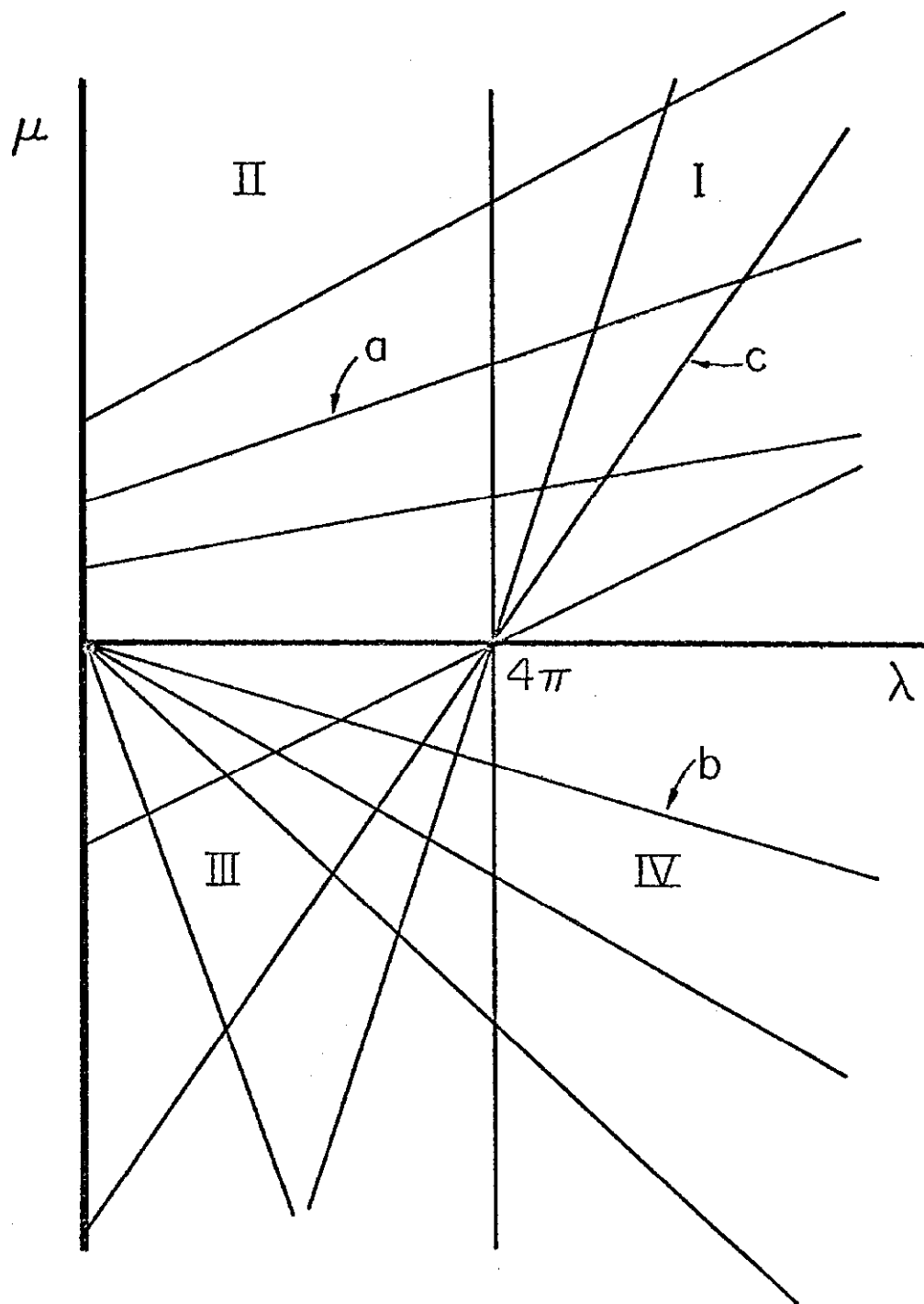
Figure 3

Figure 4

Figure   5